이방성 매질 층 내 빛 추적 기법 연구

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Optical approach for Ray Tracing in Anisotropic Media

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Abstract

In this paper, we introduce the optical approach for the calculation of optical path in the birefringence media for controlling the incident light and also, calculate the ray path in the anisotropic media. In calculation, we multi-dimensionally calculated the Poynting vector S and wave vector **k** of ordinary and extra-ordinary rays, and considered both y-axis and z-axis interfaces by using the dielectric tensor rotation method. Finally, we could achieve the exact ray path in various LC modes.

I. Introduction

Current study for the liquid crystal (LC) is requiring the ray tracing in the interface of the LC cell layer because the current LC cell is becoming to use as ray controlling device such as tunable lens. Therefore, exact calculation of ray path of the ordinary (o) and extraordinary (e) waves of the light passing though the LC cell become more important. In general, LC modes consist of many birefringence layers whose orientations of LC directors are continuously changed between neighbouring LC directors in polar and azimuth direction between two isotropic substrates. Therefore, exact ray position of the light after passing through the LC mode should be obtained after important calculation; refraction and reflection at the interface between isotropic and uniaxial birefringence layer, between inhomogeneous uniaxial to uniaxial layer. Therefore, the calculation of the refraction property as a function of the difference of the director orientation between the neighbouring LC molecules is important for simple and exact calculation. we assumed LC modes as multiple stacked birefringence layers between two glass substrates. The phase matching in birefringence to birefringence interfaces was completed at z-axis and y-axis in each grid in the LC cell by calculating the wave vector **k** and the Poynting vector S of an o-ray and an e-ray. After calculation of the vectors of rays on the z-axis interface, we checked if the rays in the grid meet the y-axis interface on the grid. In the case of the ray met the y-axis interface, we could achieved the Poynting vectors of two rays by using dielectric tensor rotation method. From the optical approach, we calculated ray path of representative LC modes; ECB. We also traced the direction of the e-ray and o-ray when the orientation of LC directors in each mode is continuously changed as a function of the applied voltage state, we control the ray path of the light in the LC modes.

Ⅱ. Theory for Calculation of Ray Path of the Light in Multi−Dimensional Interfaces

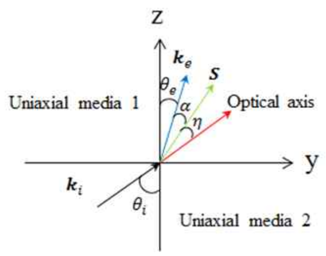
In general, LC modes exist as two multi−dimensional interfaces, which are an isotropic−uniaxial (I−U) medium and a uniaxial−uniaxial (U−U) medium. The electric fi−eld E in anisotropic media such as LC modes is not equal to the dielectric displacement vector D caused by dielectric tensor ε, which is represented at xyz−coordinate system as $= (\theta,\phi)\varepsilon R(\theta,\phi)\ ^{1}$, and $R(\theta,\phi)$ is rotational matrix and the polar angle θ and azimuth angle ϕ define the optical axis of the LC directors in modes. Assuming that the LC layer are inhomogeneously aligned along the y−z plane on each layer, we can decide the coordinate rotation matrix at y−z plane to solve the tensor ε.

Basically, incident light in birefringence media of LC mode can be divided into e−wave and o−wave. All rays of o−wave in LC layer can be calculated from the Snell's law because of isotropic properties. However, e−wave does not obey the Snell's law so we should solve the wave equation to achieve the ray path vector, which means the Poynting vector S. Figure 1(a) and 1(b) show the relation of the e−wave vector $\mathbf{k_e}$ and the Poynting vector S at anisotropic interface and boundary condition for achieving the angle θ_e. We could calculate the Poynting vector S as shown in the following equation [8, 9];

$$S = \frac{(\sin\theta_e\hat{y} + \cos\theta_e\hat{z})\sin(\eta+\alpha)}{\sin(\eta) - (\sin\theta\cos\phi + \cos\phi z)} \quad (1)$$

where, θ_e is angle between the vector $\mathbf{k_e}$ and the z−axis, and the dispersion angle α is a difference angle between the vector $\mathbf{k_e}$ and vector S. Thenη which is the angle between the optical axis of LC layer and the $\mathbf{k_e}$ could be calculated by using the equation $\cos^{-1}(\sin\theta\cos\phi\sin\theta_e + \cos\theta\cos\theta_e)$. As shown in Fig. 1, the vector S can be determined by angle η, α, and θ_e .

(a)



(b)

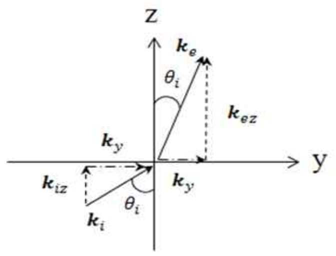


Figure 1. (a) The relation of the e−wave vector $\mathbf{k_e}$ and the Poynting vector S at uniaxial to uniaxial medium interface and (b) the boundary condition for achieving the angle θ_e.

We first consider the angle θe for the I−U and U−U medium, respectively. In the I−U interface, the angle θ_e is easily calculated by relation between $\mathbf{k_y}$ and $\mathbf{k_{ez}}$ with $\theta_e = \tan^{-1}(k_y/k_{ez})$. The vector $\mathbf{k_y}$ is simply calculated from the equation $n(\sin\theta_i)$ by boundary condition in Fig. 1(b), and $\mathbf{k_{ez}}$ is calculated by using the wave equation which is expressed into two quadratic equations [8-10]. On the other hand, the refractive index (n) at U−U interface is changed depending on the incident direction so that the vector ky could be represented as $n_{eff}(\theta, \phi)\sin\theta_i$. The n_{eff} which is effective refractive index can be calculated using the follow equation [11],

$$n_{eff}(\theta,\phi) = \frac{n_e n_o}{(n_0^2 - (n_0^2 + n_e^2)(\sin\theta\cos\phi\sin\theta_e + \cos\theta\, sin\theta_e)^2} \quad (2)$$

Then, the dispersion angle α in incident plane which is correlated by vector $\mathbf{k_e}$ and optical axis in LC layer can be achieved using the equation

$\tan^{1}(\varepsilon_e-\varepsilon_0)\tan\eta/(\varepsilon_e+\varepsilon_0\tan^2\eta))$. Finally, we can calculate the Poynting vector S of e−wave in anisotropic media from the calculated angle α and η. Figure2 shows the incident and output wave vector **k** at two interfaces. The calculated Poynting vector S of an e−wave should consider y and z−axis interface depending on the light passing through the LC layer. To solve this simple problems, we used the dielectric tensor rotation method as following equation:

$$\varepsilon = \ (\theta_r)R(\theta,\phi)\varepsilon R(\theta,\phi)^{-1}R(\theta_r)^{-1}.$$

Here, $R(\theta_r)^{-1}$ represents the electric tensor rotation matrix and angle θ_r is defined as 90° due to the orthogonality between y−axis and z−axis. Consequently, we can simply calculate the angles θ_e' and θ_o' at y−axis interface by rotating the 90○ after determining the θ_e and θ_o at z−axis interface. This dielectric tensor rotation method can provide the multi−dimensional ray tracing in birefringence medium so that the calculation of ray path of the light in all LC modes can be performed by applying this method.

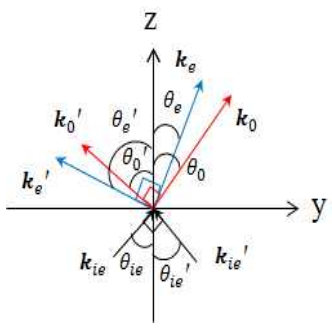


Figure 2. Illustration of rotated direction of wave vector **k**, at two interfaces.

III. Calculation for Ray Path of the Light in LC Modes

To demonstrate the proposed optical approach for calculation of ray path in birefringence media, we first calculate the propagation of rays in LC lens cell depending on the applied voltages. Figure 3 shows the cross−sectional view of the structure for tunable LC lens cell. The electrode width of both the common and the pixel line is 20 μm and distance between two electrodes is set to 60 μm, respectively. Top electrode has common electrode. The cell gap is set to 30 μm. Figure 4 shows the calculated rays of wave vector $\mathbf{k_e}$ and $\mathbf{k_o}$ in the LC lens cell at y−z interface [12, 13]. The inset in Fig. 4 presents the calculated profile of LC director (Merck, MAT−10−566, n = 0.2276, n_o = 1.5219, n_e = 1.7495, ε = 6.6) in the LC lens cell at each applied voltage. We designed the tunable LC lens using an electrically controlled birefringence (ECB) mode (the azimuth angle ϕ =0○) which has a focal length of 125 μm [14, 15].

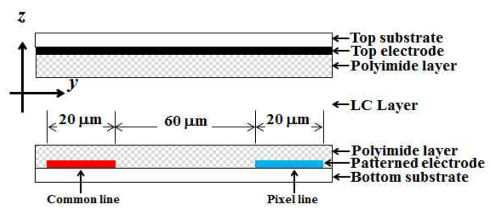


Figure 3. Schematic diagram of the cell and electrode structure for calculation of ray path in LC mode.

In the initial state in Fig. 4(a), the solid lines, which represent the extra−ordinary ray, are propagating along the z−axis in LC layer without any change of path due to the homogeneous state. After applied the voltage in LC lens cell in Fig. 4(b) rays are slightly gathering because of the symmetrical alignment of the LC directors along the z−axis. Finally, we observed that all rays in the LC lens cell at 6 V are focused at 125 μm as shown in Fig. 4(c). On the contrary, the ordinary ray which is dotted line straightly propagated without change of voltages because of the isotropic properties of LC lens cell. Additionally, we confirmed that all rays always stay on the y−axis because the azimuth angle is

0°. As a result, we can expect that the azimuth angle ϕ in LC cell is closely related to propagating direction of rays in birefringence media.

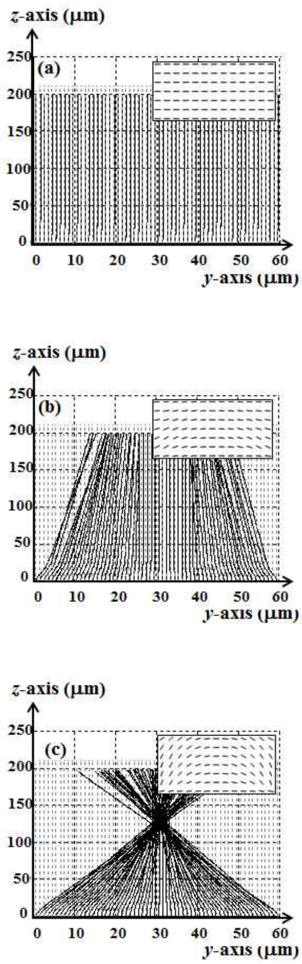


Figure 4. The calculated ray path of wave vector K_e and K_o in an electrically controlled birefringence (ECB) mode as a function of the applied voltage: (a) 0 V, (b) 4 V, and (c) 6 V

IV. Conclusion

In summary, we reported the optical approach for calculation of ray path in the anisotropic media.. To achieve the extra−ordinary rays iin the anisotropic media, we solve the wave vector **k** and Poynting vector S in I−U and U−U interfaces by using the phase matching method. We also considered both y−axis and z−axis interfaces from the dielectric tensor rotation method so that the exact calculation of ray path could be possible. From the optical approach, we calculated the propagation of rays in LC lens cell depending on the applied voltages and also demonstrated the ray focusing properties in LC lens cell without azimuth angle ϕ.

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