Numerical Modeling for the Hydrodynamic Behavior of a Charged Particle in an Electrophoretic Cell

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In this paper, we propose a numerical modeling method that can calculate the hydrodynamic behavior of a micro-sized charged particle in an electrophoretic cell. In order to simulate the electrokinetic characteristics of a particle in a real cell, we applied the electrostatic body force of the particle to the total dynamic interaction force and calculated the electrophoretic mobility so that we could calculate the dynamic response of the particle in a viscous fluid. We compared the calculated result with the experimental result of a horizontal switching electrophoretic cell for verification.

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I. INTRODUCTION

Recently, many techniques to realize electronic paper [1, 2] have emerged using electrophoretic displays, electrowetting displays, liquid crystal reflective displays, electrochromic, Gyricon, and others [3–5]. In particular, works on displays based on the principles of electrophoresis began in the 1970s, and many studies and experiments have been performed in order to achieve high electro-optical characteristics. The electrokinetic properties of latex particles used as a suspending medium were presented [6]. The other efforts for electrophoretic display (EPD) cells were performed by using in-plane switching type cells [7], wall/post electrode patterns [8], liquid crystals [9], and so on.

Similar to the other display devices, the EPD also requires a numerical approach to calculate the electro-optical properties of the cell before we performing experiments because understanding the dynamic behavior of the particle helps in recognizing the extent to which the electro-optical characteristics can be obtained. Recently, several numerical methods have been studied in an effort to simulate colloidal dispersions. If the dynamical behavior of the charged colloidal suspension is to be understood, one of the most important studies on EPD cells is to calculate the electrokinetic property at the fluid/solid interfaces in order to avoid the complexity of the calculation [10,11].

In a previous paper, Tanaka and Araki proposed a simulation method that could find the electrophydrodynamic property at the fluid/solid interfaces by using a multi-phase system. They assumed that the solid and the fluid media could be handled identically, and they expected the particle in the fluid to be hardly deformable in the calculations by forcing the solid to have a much higher viscosity than the fluid. As a result, they could achieve acceptable results with a fast calculation [12]. Kodama et al. presented a simulation result for the dynamics of a charged colloidal suspension by using Tanaka’s method so as to incorporate the electrohydrodynamic interactions between the three relevant elements of the charged colloidal suspension [13] (i.e., colloidal particles, ion clouds and liquid). They assumed that the charged colloid consist of the non-electrostatic body force and the attached ions. However, as for the simulation of the behavior of the charged particles in the EPD device, we have to consider the electrostatic body force of the charged particle to calculate the electrophoretic mobility of the charged particle.

In this paper, we propose a numerical method to model the electrohydrodynamic behavior of the charged particle in a 2-dimensional electrophoretic cell. We consider the electrostatic body force of a charged particle in a viscous fluid and calculate the electrophoretic mobility. In our calculations, we ignored the ion force because, generally, the electrophoresis force in the EPD cell dominates the movement of the particle compared to the ion force.

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The electric potential distribution in the cell is obtained by considering the boundary condition at the interface of the patterned glass and the fluid in the EPD cell. We also applied the two-phase approach at the solid/liquid interface for fast and exact results. The calculation results were compared to the experimental results for a patterned horizontal switching EPD cell for verification.

II. CALCULATION OF THE DYNAMICAL MOTION OF A CHARGED PARTICLE IN A VISCOUS FLUID

1. Potential Distribution in an EPD Cell

Figure 1 shows the typical cell structure of a horizontal switching EPD cell. The cell consists of two glasses for the substrate, the fluid and the charged particle. The cell contains electrodes, and we apply the voltages \( \psi_1 \) and \( \psi_2 \) to the two electrodes for switching. The parameters \( \alpha \) and \( d \) represent the radius of the charged particle and the cell gap. Table 1 shows the parameters used for the calculations and the experiments.

First of all, electrostatic potential distribution in each layer should be calculated in order to calculate the associated electrophoretic flows and can be performed using Poisson’s equation \( \rho_c = \text{div}(\epsilon \text{grad} \psi) \). Here, \( \epsilon \) is the dielectric permittivity of the medium, \( \epsilon_0 \) is the permittivity of vacuum and \( \rho_c \) is the charge density of the particle. We calculate the electric potential distribution by using a successive over-relaxation (SOR) method to solve the Poisson equation:

\[
\psi_{i,j} = \psi_{i,j} + \omega (\psi_{i,j} - \psi_{i,j}^{(n)}),
\]

where \( \omega \) is an over-relaxation constant. The variable \( \psi_{i,j} \) indicates the values obtained in the previous time step. The parameters \( i \) and \( j \) represent the grid position numbers in the electrophoretic cell. We use the optimized value 1.45 for \( \omega \).

For the calculation of the potential distribution, we need to consider the boundary conditions of two interfaces, the fluid/glass and the glass/air interfaces, of the cell. At the fluid/glass interface, we can simply use the continuous normal \( z \)-component of the electric flux density \( D \). The \( z \)-components of \( D \) in the fluid and the glass are as follows:

\[
D_z^{\text{fluid}} = -\varepsilon_0 \left( \frac{\varepsilon_{\text{fluid}}}{\varepsilon_{\text{fluid}}} \frac{d\psi}{dz} \right), \\
D_z^{\text{glass}} = -\varepsilon_0 \left( \frac{\varepsilon_{\text{glass}}}{\varepsilon_{\text{glass}}} \frac{d\psi}{dz} \right),
\]

where \( \varepsilon_{\text{fluid}} \) is the dielectric permittivity of the fluid and \( \varepsilon_{\text{glass}} \) is the dielectric permittivity of the glass. To satisfy the boundary condition, \( D_z^{\text{fluid}} \) should be the same as \( D_z^{\text{glass}} \) at the fluid/glass interface. At the glass/air interface of the cell, we apply the boundary condition \( d\psi/dz = 0 \) at \( z = 4d \) to reduce the error rate of the calculations [14].

2. Hydrodynamic Model

Adopting a continuum assumption, electrophoretic flow can be modeled using the Navier-Stokes equations with the electric body force \( F \). The Navier-Stokes equations with the electric body force are expressed as,

\[
\nabla \cdot \vec{V} = 0, \\
\rho \left( \frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} \right) = -\nabla p + \mu \nabla \left( (\nabla \vec{V}) + (\nabla \vec{V})^T \right) + \vec{F},
\]

where \( \vec{V} \) denotes the velocity vector comprising \( u \) (x-direction) and \( v \) (z-direction), \( p \) is the static pressure, \( \rho \) is the fluid density and \( \mu \) is the fluid viscosity. We can also set the second term \( \nabla \cdot \vec{V} \) of an incompressible fluid to be zero (Stokes approximation) because the Reynolds number can be quite small for aqueous suspensions of colloids of sub-micron size [12].

In Eq. (4), we assume that the density of the solid particle is the same as that of the liquid in order to satisfy the requirements of a two-phase density system. The two-phase system for separating the phase between the fluid and the particle can be achieved using the difference in the viscosity. For the phase separation, we use the following equation, which depends on the central position of the particle [13]:

\[
\phi_{i,j} = 0.5 \left[ 1 + \tanh \left( \frac{\alpha - \sqrt{(x_{i,j} - x_p)^2 + (y_{i,j} - y_p)^2}}{\xi} \right) \right],
\]

where \( (x_p, y_p) \) is the position of the solid particle’s center, and \( \xi \) denotes the interface width. From this phase function, the solid area and the fluid are assigned values of 1 or 0, so that we can classify the area between the solid and the fluid for calculation.
and define the position of the solid particle's field $E$ as the ratio of the velocity $u$, which is the net electric charge density of the particle, and the electrophoretic mobility $\mu$ of the particle, the electrophoretic mobility $\mu_{ep}$ is defined as the ratio of the velocity $u_{ep}$ to the applied electric field $E$, which is $u_{ep} = \mu_{ep} E$. As for the electrophoretic mobility of a charged particle, it can be described as $\mu_{ep} = \rho_p/6\pi\mu_f \tau$ [15], $\tau$ represents the Stokes’ radius of the particle [16], which is same as $\alpha$ in our system. From this, we can simply calculate the effective volume of the particle.

### III. CALCULATED RESULTS AND DISCUSSION

We employ a uniform, staggered grid system in order to solve Eqs. (3) - (6) and an explicit Euler integration for calculating the time evolution. Figure 2 shows a microphotograph example of the EPD cell at 100 V, and the details of the experiment are shown in Table 1. A charged particle will be attracted toward the electrode with opposite electric polarity and will be moved to an opposite electrode whenever the electric polarity of the electrode is changed. Figure 2 (a) shows the particle distribution, that is spread out on the wide electrode for the dark state. On the other hand, the particles are distributed on the narrow electrode for the bright state, as shown in Fig. 2(b).

Figure 3 shows the calculated results for the potential distribution of the fluid layer when we apply 100 V for $\psi_2$ and 0 V for $\psi_1$. The curved arrow lines imply the directions of the electric field in the EPD cell. From the calculated result, we can simulate the motion of the particle due to the electrophoretic force.

Figure 4 shows that the 2-dimensional calculated result for the electrophoretic flow process of the charged particle as a function of time. At the fluid/glass interface, the no-slip boundary condition $V = 0$ is employed, and a periodic boundary condition is applied. In our simulation, we assume that the diffusion of the particle in the fluid is much slower than the drift motion, so we ignore the velocity of the particle due to diffusion. In addition, we also assumed no random Brownian motions of particles for simplicity [12]. Figure 4 shows the calculated result of the movement of the particle due to the electric force. Before the voltage is applied, we put a particle on the center position of the narrow electrode. Figure 4(a) shows the position of the charged particle

### Table 1. Parameters of the EPD cell for the calculation and the experiment.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle radius ($\alpha$)</td>
<td>3 $\mu$m</td>
</tr>
<tr>
<td>Charge density of material ($\rho_e$)</td>
<td>-8 $\mu$C/g</td>
</tr>
<tr>
<td>Viscosity of dispersant fluid (silicon oil) ($\mu_f$)</td>
<td>10 cs</td>
</tr>
<tr>
<td>Dielectric constant of fluid (silicon oil) ($\varepsilon_f$)</td>
<td>2.6</td>
</tr>
<tr>
<td>Reynolds number ($R_e$)</td>
<td>1</td>
</tr>
<tr>
<td>Interface width ($\xi$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Weight of particle and fluid</td>
<td>10 wt%</td>
</tr>
<tr>
<td>Cell gap ($d$)</td>
<td>60 $\mu$m</td>
</tr>
<tr>
<td>Width of left/right electrode line</td>
<td>20 $\mu$m/80 $\mu$m</td>
</tr>
<tr>
<td>Distance between bottom two electrodes</td>
<td>10 $\mu$m</td>
</tr>
<tr>
<td>Distance between bottom electrode and end position of cell</td>
<td>5 $\mu$m</td>
</tr>
</tbody>
</table>
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Fig. 4. (Color online) Process snapshots of the electrophoretic flow. The lines in the cell imply the velocity field in the fluid layer (a) at 168.86 ms and (b) at 253 ms.

during the voltage application. We can observe that the charged particle follows the direction of the electric field in the cell. Figure 4(b) shows the negative charged particle finally reaching the wide electrode. As for the micro-scaled particle associated with the EPD application, the Reynolds number is getting so small that laminar flow should be shown in the cell during the movement. From Fig. 4, we can easily observe the velocity field to explain the micro-scaled flow which, characteristics imply laminar flow. From the calculation, we obtained 111.56 ms as the time required for the charged particle to pass the end of the opposite electrode (i.e., the switching time of the cell).

In general, the dynamic response time of each particle in the EPD cell on the start electrode is different because particles closer to the opposite electrode obviously respond more quickly than particles farther from the opposite electrode. Therefore, the optical response of the EPD cell strongly depends on the particles that are far from the target electrode.

Figure 5 shows the calculated dynamical behavior of the particle that is the farthest from the opposite electrode on the center position of the start electrode. Figures 5(a) and (b) show the motion contours of the particle from the left electrode to the right electrode as a function of the moving time, which provides the falling time of the EPD cell, and the motion contours of the particle from the right electrode to left electrode, which also provides the rising time. The exact optical response time of the EPD cell can be calculated by observing the time necessary for the particle to pass the end of each electrode. From Fig. 5, we can simply calculate 380.67 ms for total optical response time, which consists of 111.56 ms for the falling time and 269.11 ms for the rising time. Similarly, our calculation at 200 V under the same cell condition yielded 123.81 ms for the total optical response time, which consist of 40.6 ms for the falling time and 83.21 ms for the rising time. Figure 6 shows the experimental result for verification. The experimental results at 100 V and 200 V shows 370 ms and 128 ms for the total optical response time, respectively. This result shows very good agreement with the calculated results.

IV. CONCLUSIONS

In summary, we proposed a numerical modeling method for EPD cells by applying the electric body force of the particle to the electrokinetic calculation. We calculated the velocity of the particle by solving the electrokinetic equation and calculating the electrophoretic mobility of the particle. Verification of the calculation was performed by comparing the calculated result to the experimental result.
ACKNOWLEDGMENTS

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