Numerical solutions for LC optics

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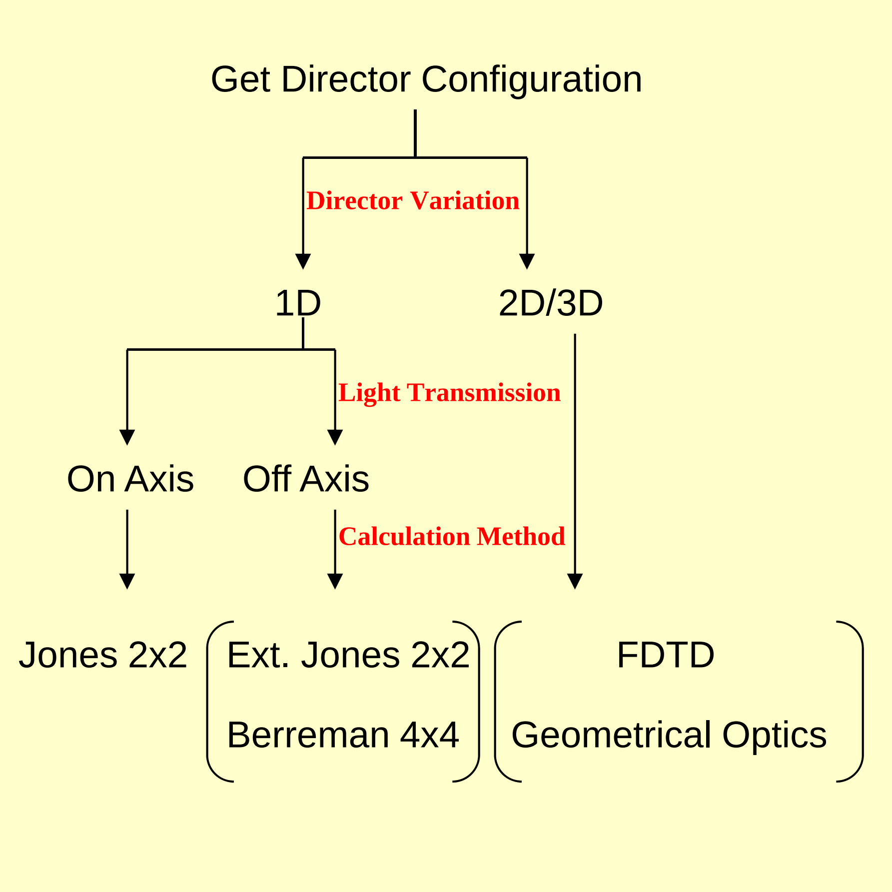
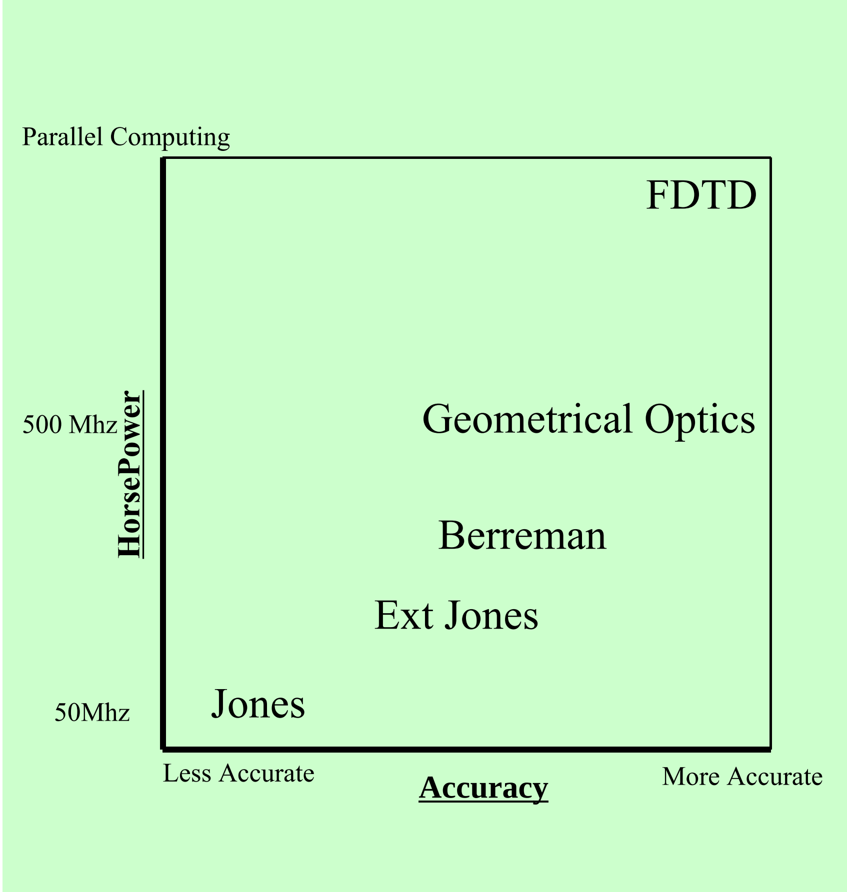
Outline:

- ***2×2 Jones matrix method***
- 2×2 extended Jones matrix method
- 4×4 matrix method (Berreman, Wohler, Abdulhalim)
- Geometric optics (Poin'care sphere method)
- Finite Difference Time Domain method

Optical Modeling Outline





2×2 Jones matrix method

- Uses 2x2 matrices to define optical elements such as polarizers & retarders.
- A two component column vector represents the polarization state of light.
- Interaction of above two matrices gives light throughput.
- Only for on-axis light transmission.
- Ignores interference effects in system.
- Simple to use for like systems.

• Jones vector representation

$$\vec{E}(z,t) = \mathrm{Re}(\vec{A}e^{i(\omega t - kz)})$$ Complex representation

$$\vec{A} = Ax\, e^{i\delta x}\,\overrightarrow{ax} + Ay\, e^{i\delta y}\,\overrightarrow{ay}$$

$$E_x = A_x \cos(\omega t - kz - \delta_x)$$
$$E_y = A_y \cos(\omega t - kz - \delta_y)$$

$$\longrightarrow \quad J = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix}$$

Complex amplitude

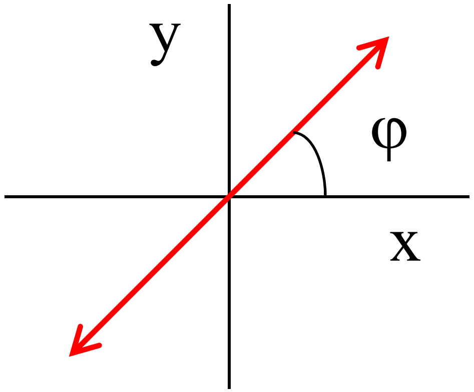
$$J = \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix} \xrightarrow{\text{Normalization}} J^* \cdot J = 1$$

*J** : complex conjugation

$$\therefore \; J^* \cdot J = \begin{pmatrix} A_x e^{-i\delta_x} & A_y e^{-i\delta_y} \end{pmatrix} \begin{pmatrix} A_x e^{i\delta_x} \\ A_y e^{i\delta_y} \end{pmatrix} = A_x^{\ 2} + A_y^{\ 2} = 1$$

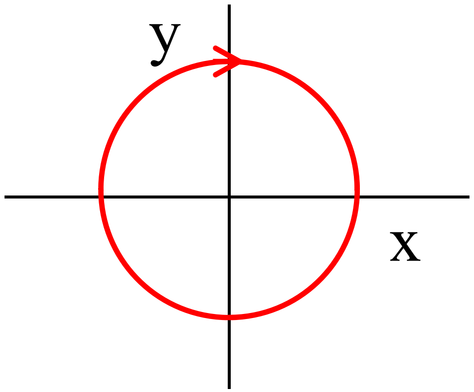
– *Linear polarization* ($\delta_x = \delta_y = 0$)

$$J/(J^* \cdot J)^{1/2} = \frac{1}{\sqrt{A_x^{\ 2} + A_y^{\ 2}}} \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

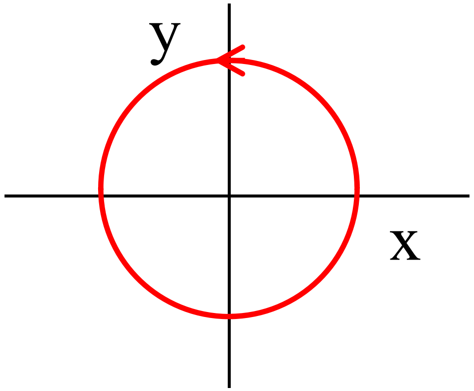


– *Circular polarization* ($A_x = A_y$, $\delta_y = \delta_x \pm \pi/2$, if $\delta = \delta_y - \delta_x$)

$$J/(J^* \cdot J)^{1/2} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{j0} \\ e^{i\pm\pi/2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm i \end{pmatrix}$$



$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$



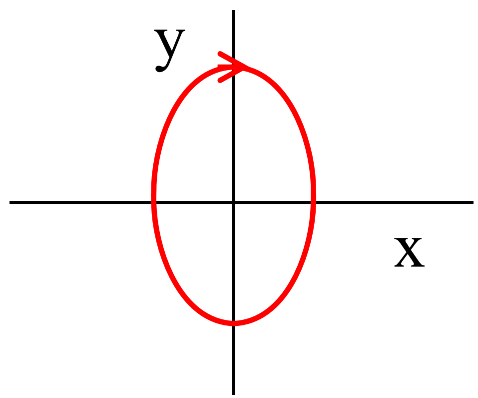
$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

– *Elliptic polarization* ($\delta = \delta_y - \delta_x$)

$$J/(J^* \cdot J)^{1/2} = \frac{1}{\sqrt{A_x^{\,2} + A_y^{\,2}}} \begin{pmatrix} A_x e^{i0} \\ A_y e^{i\delta} \end{pmatrix} = \begin{pmatrix} \cos\varphi \\ e^{i\delta} \sin\varphi \end{pmatrix}$$

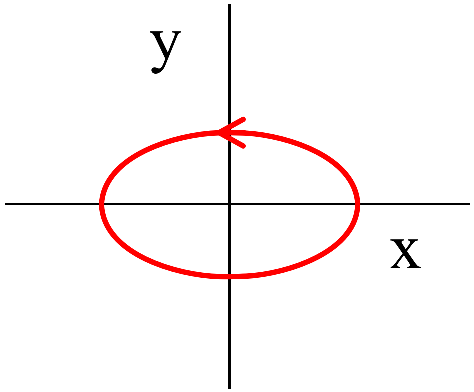
– if ($A_y = 2A_x$, $\delta = \pi/2$)

$$J/(J^* \cdot J)^{1/2} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 \\ 2i \end{pmatrix}$$



– if ($2A_y = A_x$, $\delta = -\pi/2$)

$$J/(J^* \cdot J)^{1/2} = \frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ -i \end{pmatrix}$$



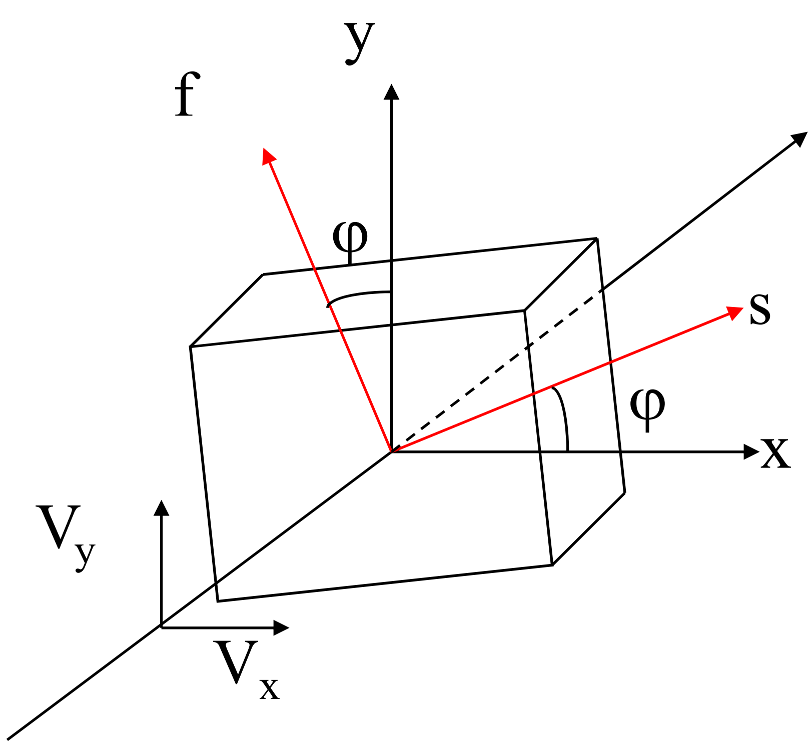
• Jones calculus for wave propagation in a anisotropic media

– We are going to mention on only an uniaxial media

– Light passing through media consists of two eigenmodes, which are ordinary and extra ordinary wave : slow and fast axis

– We ignore all reflections

- Jones matrix formulation



Incident light beam *V*

$$V = \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

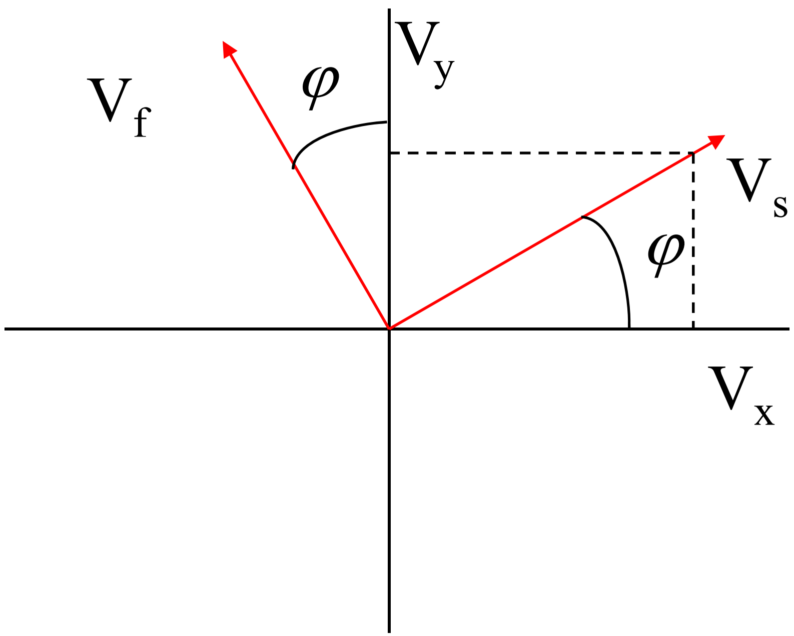
x, y, z : laboratory axis

s, f : slow and fast axis

– *Eigenwaves of the crystal*

$$V_s = V_x \cos\varphi + V_y \sin\varphi$$
$$V_f = -V_x \sin\varphi + V_y \cos\varphi$$

$$\begin{pmatrix} V_s \\ V_f \end{pmatrix} = \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$



– *Emerging beam after phase delay*

$$V_s' = e^{i(k_s l)} V_s = e^{i(\frac{2\pi}{\lambda} n_s l)} V_s$$

$$V_f' = e^{i(k_f l)} V_f = e^{i(\frac{2\pi}{\lambda} n_f l)} V_f$$

K_s : wave vector by n_s

K_f : wave vector by n_f

We define *phase shift* Γ, *mean absolute phase change* Φ

$$\Gamma = k_s - k_f = (n_s - n_f)\frac{2\pi}{\lambda} l \rightarrow (n_e - n_o)\frac{2\pi}{\lambda} d = \frac{2\pi}{\lambda} \Delta nd$$

$$\Phi = (n_s + n_f)\frac{\pi}{\lambda} l$$

Then,

$$\begin{pmatrix} V_s' \\ V_f' \end{pmatrix} = e^{-i\Phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \begin{pmatrix} V_s \\ V_f \end{pmatrix}$$

– Transforming back to *xy* coordinate system

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} V_s' \\ V_f' \end{pmatrix}$$

$$= \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} e^{-i\Phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

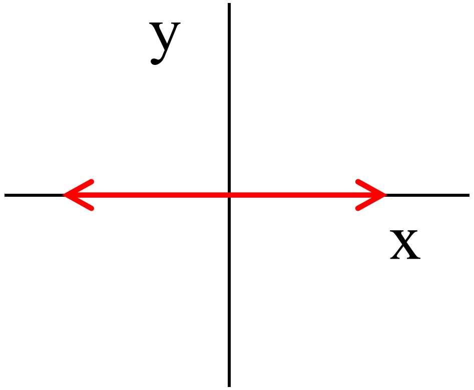
$$= R(-\varphi) W_0 R(\varphi) \begin{pmatrix} V_x \\ V_y \end{pmatrix} = W \begin{pmatrix} V_x \\ V_y \end{pmatrix}$$

$R(\varphi)$: rotation matrix
W_0 : Jones matrix for retarder

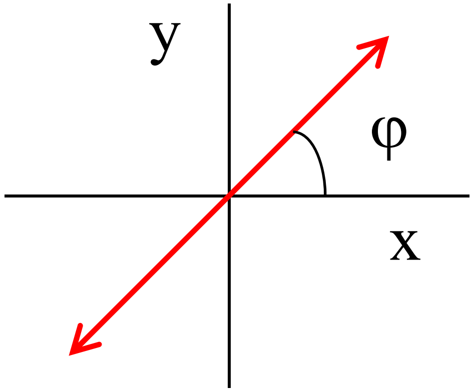
$$R(\varphi) = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$$

$$W_0 = e^{-i\Phi} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \quad e^{-i\Phi} \textit{ is negligible}$$

– *Polarizer* (*Γ=0*)

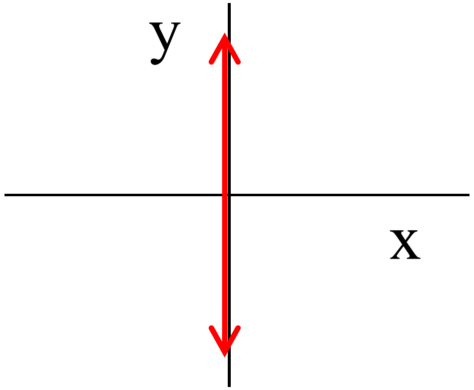


$$\rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$



$$\rightarrow \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\varphi & \cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{pmatrix}$$



$$\rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

– *Retarder* *(Γ≠0)*

1. half-wave plate *(Δnd= π/2)* ⟶ $\Gamma = \frac{2\pi}{\lambda}\Delta nd = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi$

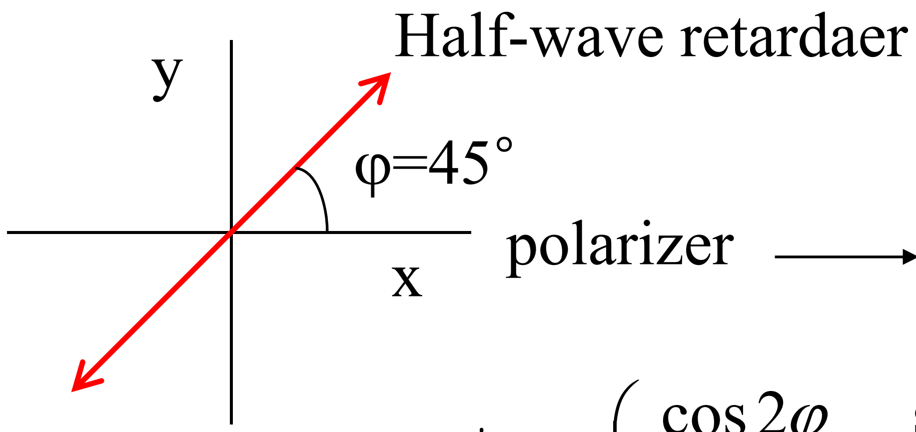
$$W = R(-\varphi)W_0R(\varphi)$$

$$= \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} e^{-i\Gamma/2} & 0 \\ 0 & e^{i\Gamma/2} \end{pmatrix} \begin{pmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{pmatrix}$$

$$= -i\begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix}$$

: 2φ rotator

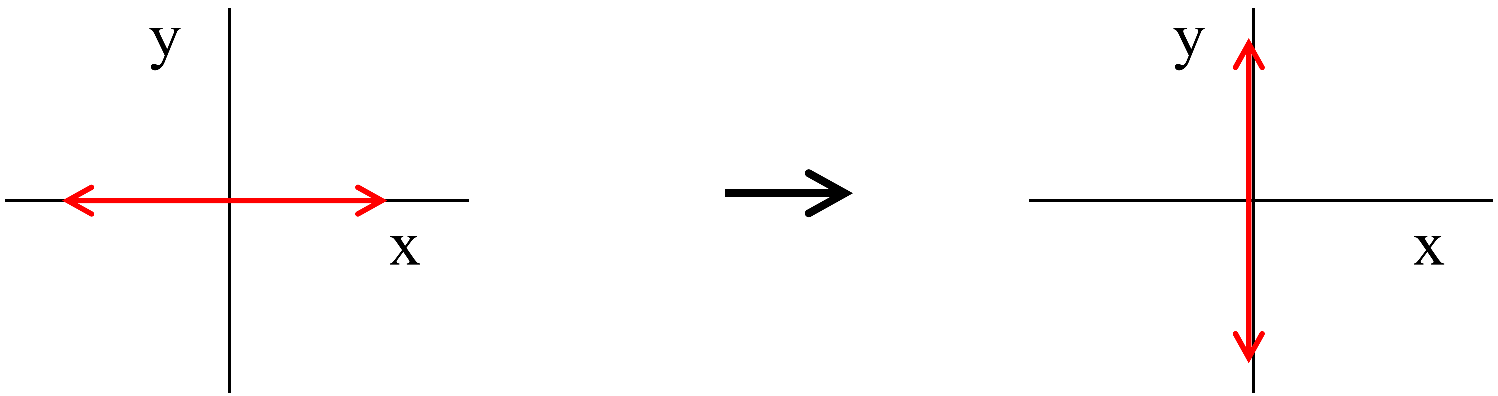
If



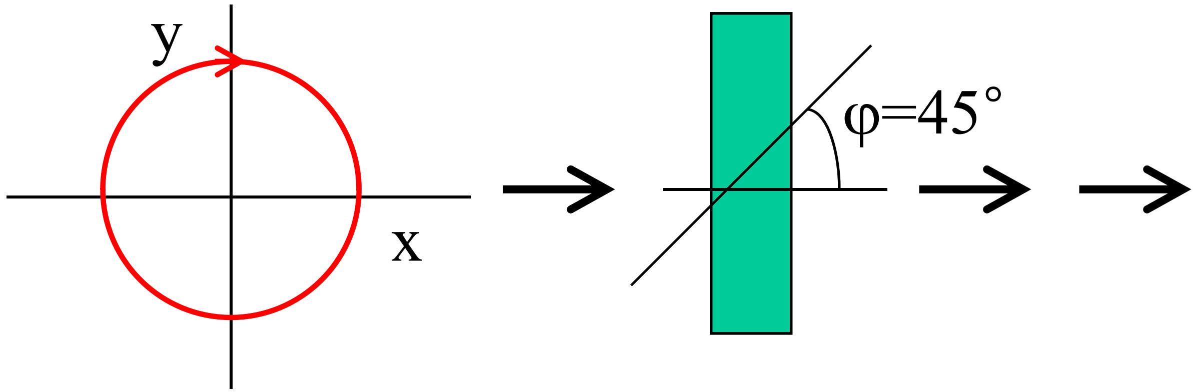
polarizer $\longrightarrow$ $P_o = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$V' = -i\begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= -i\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



If



?

Clockwise circular plarization

Half-wave retardaer

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} \qquad -i\begin{pmatrix}\cos 2\varphi & \sin 2\varphi\\ -\sin 2\varphi & \cos 2\varphi\end{pmatrix}$$

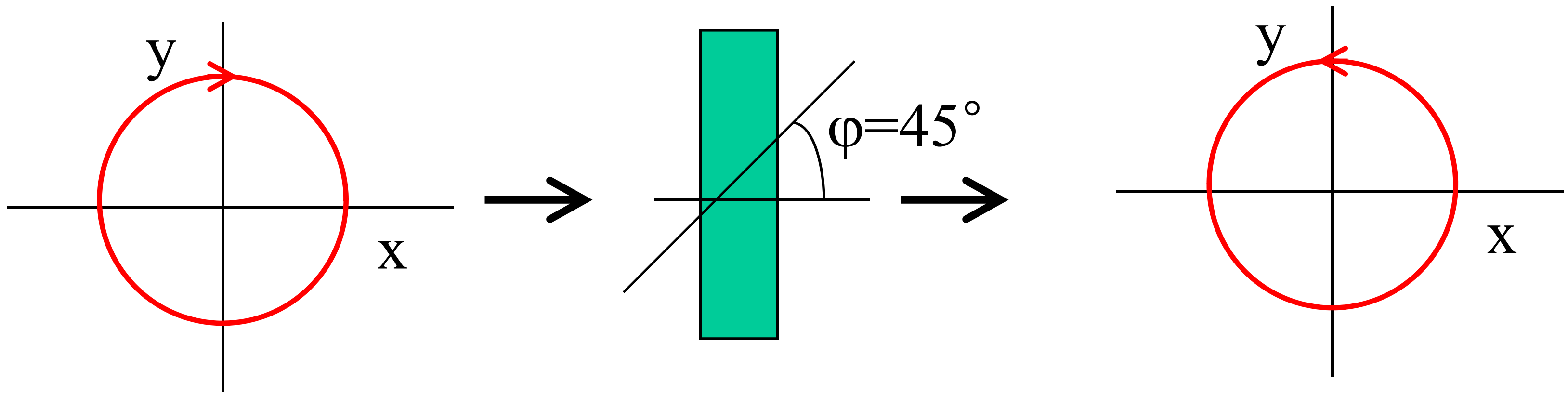
$$V' = -i\begin{pmatrix}\cos 2\varphi & \sin 2\varphi\\ -\sin 2\varphi & \cos 2\varphi\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$$

← countclockwise circular plarization

Therefore,

If



Clockwise circular plarization

Half-wave retardaer

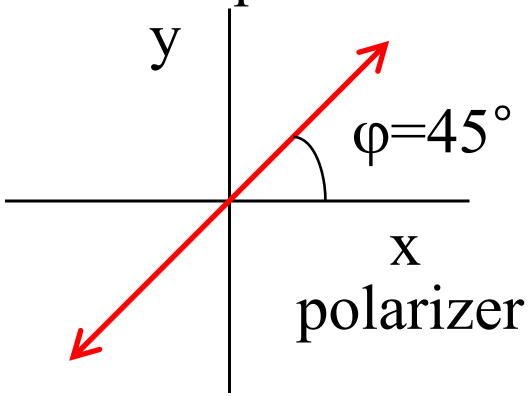
countclockwise circular plarization

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} \qquad -i\begin{pmatrix}\cos 2\varphi & \sin 2\varphi\\ -\sin 2\varphi & \cos 2\varphi\end{pmatrix} \qquad \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$$

2. quarter-wave plate *(Δnd=π/4)* ⟶ $\Gamma = \frac{2\pi}{\lambda}\Delta nd = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$

$$W = R(-\varphi)W_0R(\varphi)$$
$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 - i\cos 2\varphi & -i\sin 2\varphi \\ -i\sin 2\varphi & 1 + i\cos 2\varphi \end{pmatrix}$$

quarter-wave retardaer



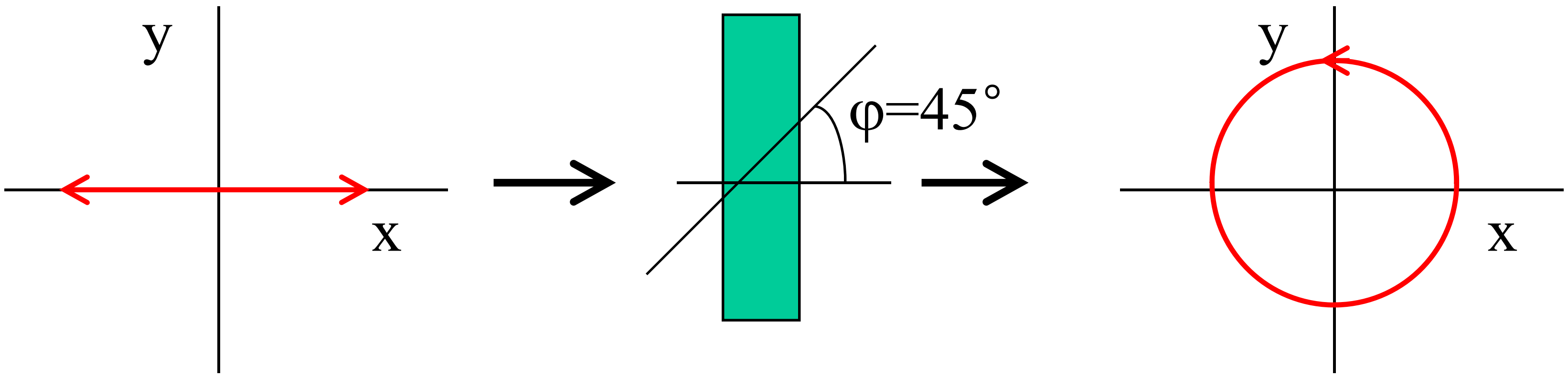
polarizer

→ **?**

$$V' = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 - i\cos\frac{\pi}{2} & -i\sin\frac{\pi}{2} \\ -i\sin\frac{\pi}{2} & 1 + i\cos\frac{\pi}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix} \longleftarrow$$

countclockwise circular plarization



– intensity transmission

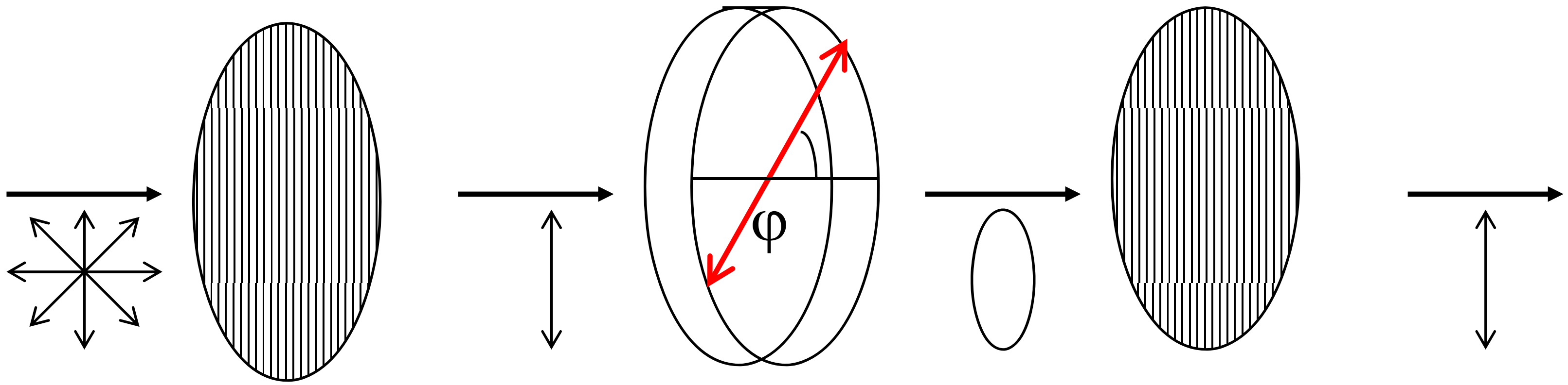
$$E = \begin{pmatrix} E_x \\ E_y \end{pmatrix} \rightarrow I = E^* \cdot E = |E_x|^2 + |E_y|^2$$

Emerging beam electric field *E'*

$$E' = \begin{pmatrix} E_x' \\ E_y' \end{pmatrix}$$

intensity transmission *T* $$T = \frac{|E_x'|^2 + |E_y'|^2}{|E_x|^2 + |E_y|^2}$$

– A retarder with parallel polarizers



$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} \qquad -i\begin{pmatrix}\cos 2\varphi & \sin 2\varphi\\ -\sin 2\varphi & \cos 2\varphi\end{pmatrix} \qquad \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

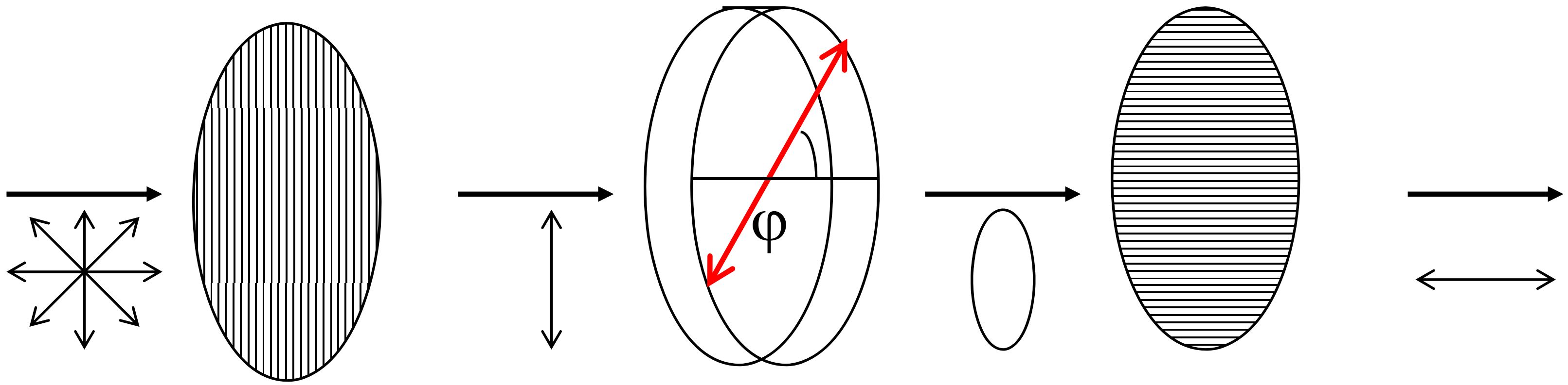
$$\mathbf{E}' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\frac{\Gamma}{2} & -i\sin\frac{\Gamma}{2} \\ -i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \cos\dfrac{\Gamma}{2} \end{pmatrix}$$

Therefore,

$$I = \frac{1}{2}\cos^2\frac{\Gamma}{2} = \frac{1}{2}\cos^2[\frac{\pi(n_e - n_o)d}{\lambda}]$$

– A retarder with crossed polarizers



$$\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} \qquad -i\begin{pmatrix}\cos 2\varphi & \sin 2\varphi\\-\sin 2\varphi & \cos 2\varphi\end{pmatrix} \qquad \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$$

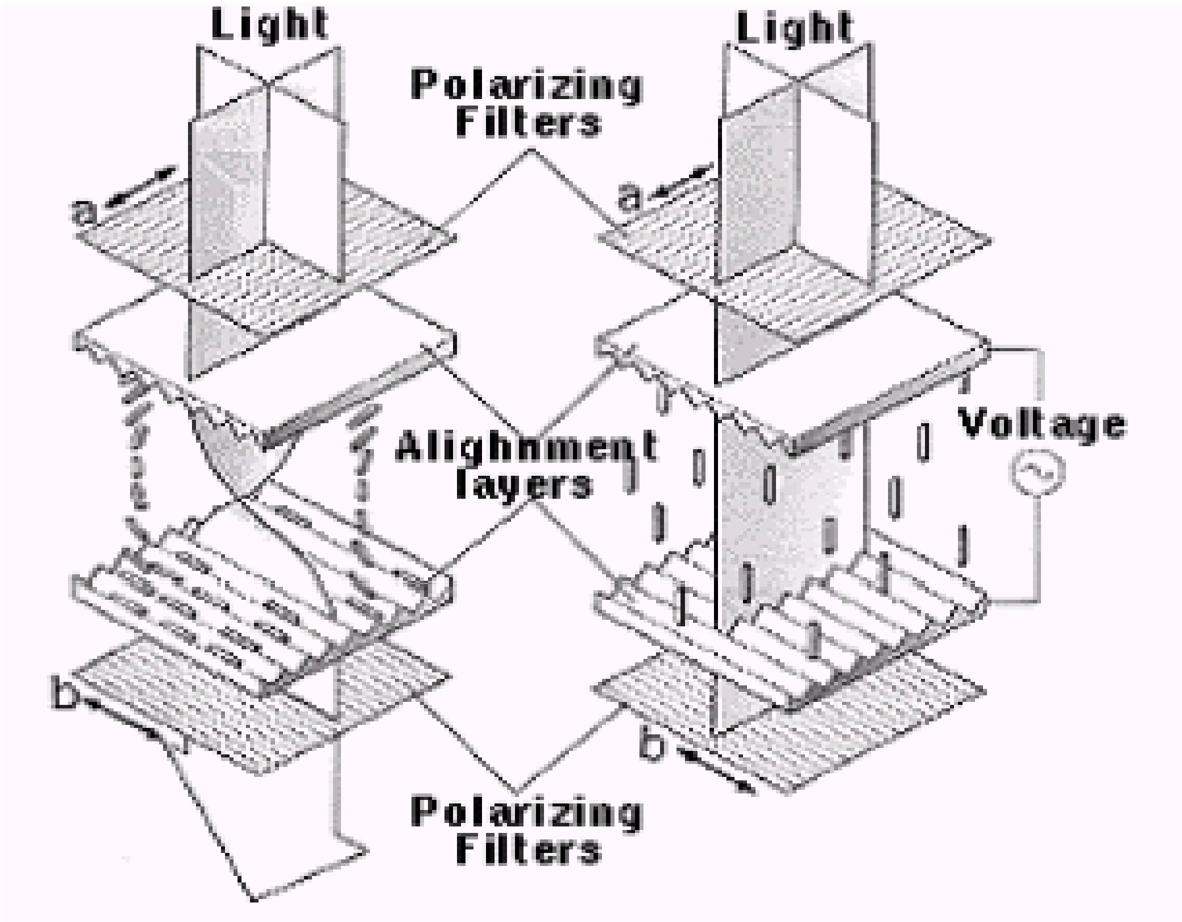
$$\mathbf{E}' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \cos\frac{\Gamma}{2} & -i\sin\frac{\Gamma}{2} \\ -i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= -\frac{1}{\sqrt{2}} \begin{pmatrix} \sin\dfrac{\Gamma}{2} \\ 0 \end{pmatrix}$$

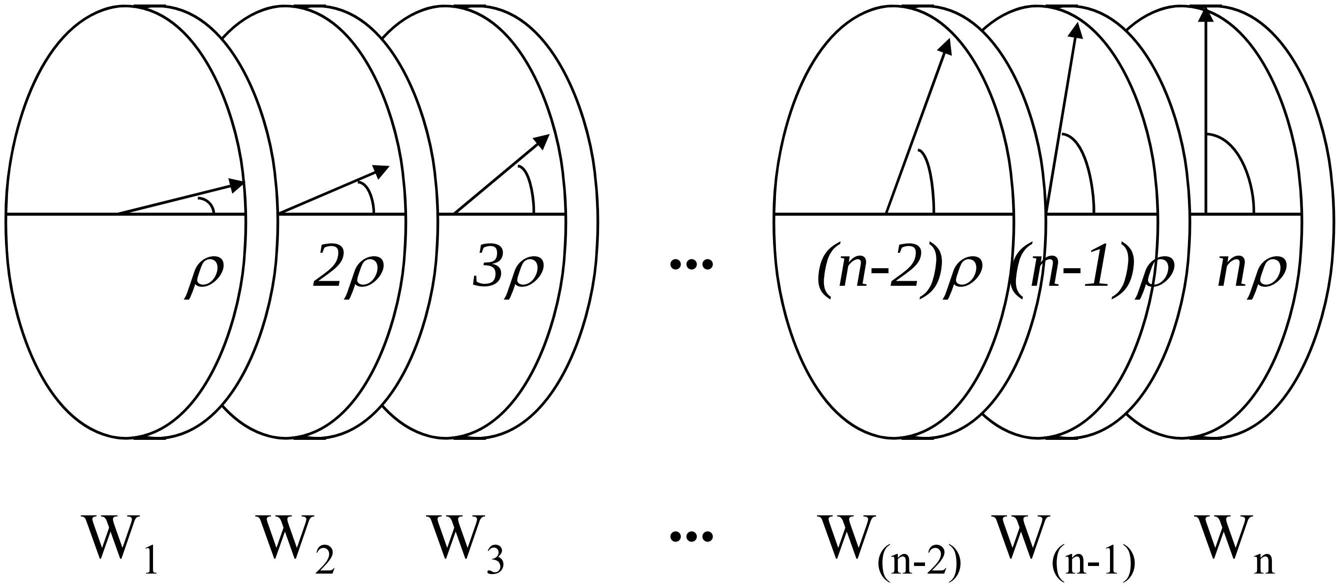
Therefore,

$$I = \frac{1}{2}\sin^2\frac{\Gamma}{2} = \frac{1}{2}\sin^2[\frac{\pi(n_e - n_o)d}{\lambda}]$$

– light propagation in twisted nematic cell



– light propagation modeling in twisted anisotropic media



$$
\begin{aligned}
M &= W_n W_{(n-1)} W_{(n-2)} \cdots W_3 W_2 W_1 \\
&= R(-n\rho) W_0 R(n\rho) R(-(n-1)\rho) W_0 R((n-1)\rho) \cdots \\
&\quad R(-2\rho) W_0 R(2\rho) R(-\rho) W_0 R(\rho) \\
&= \prod_{m=1}^{n} R(-m\rho) W_0 R(m\rho)
\end{aligned}
$$

Using identities property below

$$R(\rho_1)R(\rho_2) = R(\rho_1 + \rho_2)$$

therefore,

$$M = R(-\Phi)\left[W_0 R(\frac{\Phi}{n})\right]^n \qquad \Phi = n\rho : twist\ \ angle$$

$$= R(-\Phi)\begin{pmatrix} \cos(\frac{\Phi}{n})e^{-i\Gamma/2n} & \sin(\frac{\Phi}{n})e^{-i\Gamma/2n} \\ -\sin(\frac{\Phi}{n})e^{-i\Gamma/2n} & \cos(\frac{\Phi}{n})e^{-i\Gamma/2n} \end{pmatrix}^n$$

Using chebyshev's identity

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^n = \begin{pmatrix} \dfrac{A\sin mZ - \sin(m-1)Z}{\sin Z} & B\dfrac{\sin mZ}{\sin Z} \\ C\dfrac{\sin mZ}{\sin Z} & \dfrac{D\sin mZ - \sin(m-1)Z}{\sin Z} \end{pmatrix}$$

where, $Z = \cos^{-1}(\frac{1}{2}(A+D))$

therefore,

$$M = \begin{pmatrix} \cos\Phi & -\sin\Phi \\ \sin\Phi & \cos\Phi \end{pmatrix} \begin{pmatrix} \cos\mathrm{X} - i\dfrac{\Gamma}{2}\dfrac{\sin\mathrm{X}}{\mathrm{X}} & \Phi\dfrac{\sin\mathrm{X}}{\mathrm{X}} \\ -\Phi\dfrac{\sin\mathrm{X}}{\mathrm{X}} & \cos\mathrm{X} + i\dfrac{\Gamma}{2}\dfrac{\sin\mathrm{X}}{\mathrm{X}} \end{pmatrix}$$

$$= R(-\Phi)\begin{pmatrix} \cos X - i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} & \Phi\dfrac{\sin X}{X} \\ -\Phi\dfrac{\sin X}{X} & \cos X + i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} \end{pmatrix}$$

$$\text{where} \quad X = \sqrt{\Phi^2 + \left(\frac{\Gamma}{2}\right)^2}$$

– Waveguiding in TN cell

In the principle coordinate system

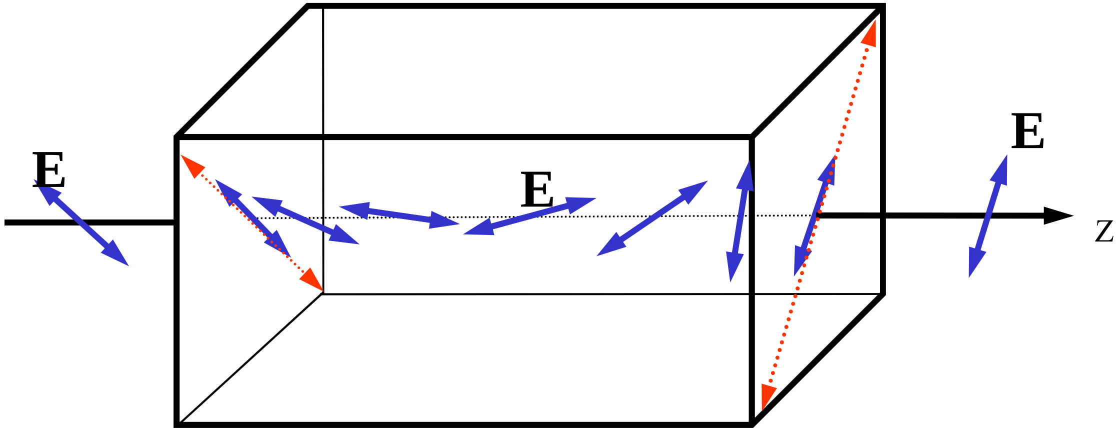
$$\begin{pmatrix} V_e' \\ V_o' \end{pmatrix} = \begin{pmatrix} \cos X - i\frac{\Gamma}{2}\frac{\sin X}{X} & \phi\frac{\sin X}{X} \\ -\phi\frac{\sin X}{X} & \cos X + i\frac{\Gamma}{2}\frac{\sin X}{X} \end{pmatrix} \begin{pmatrix} V_e \\ V_o \end{pmatrix}$$

As for e-mode wave $\begin{pmatrix} V_e \\ V_o \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, and very high $\Delta nd >> \Phi$

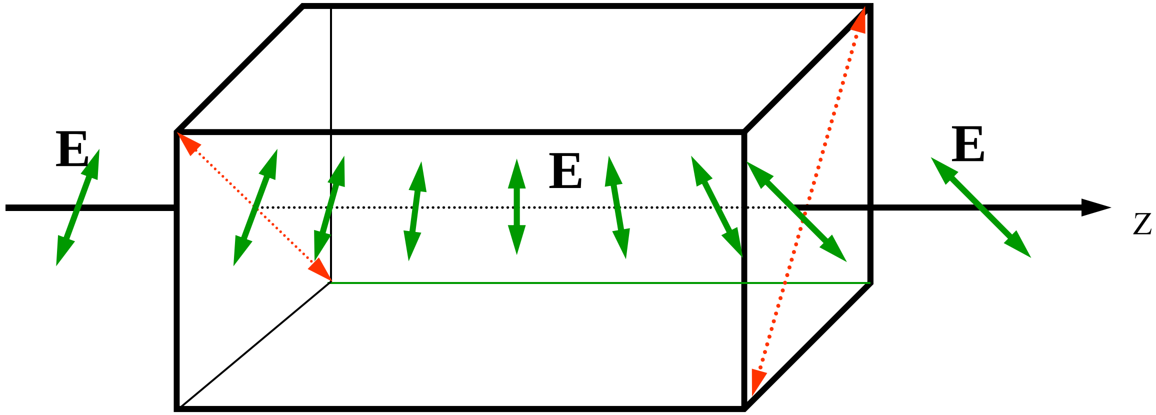
$$\begin{pmatrix} V_e' \\ V_o' \end{pmatrix} = \begin{pmatrix} \cos X - i\frac{\Gamma \sin X}{2X} \\ -\phi\frac{\sin X}{X} \end{pmatrix} \approx \begin{pmatrix} e^{-i\Gamma/2} \\ 0 \end{pmatrix}$$

$\longrightarrow$ Emerging electric field is parallel to the local director

Waveguiding!!

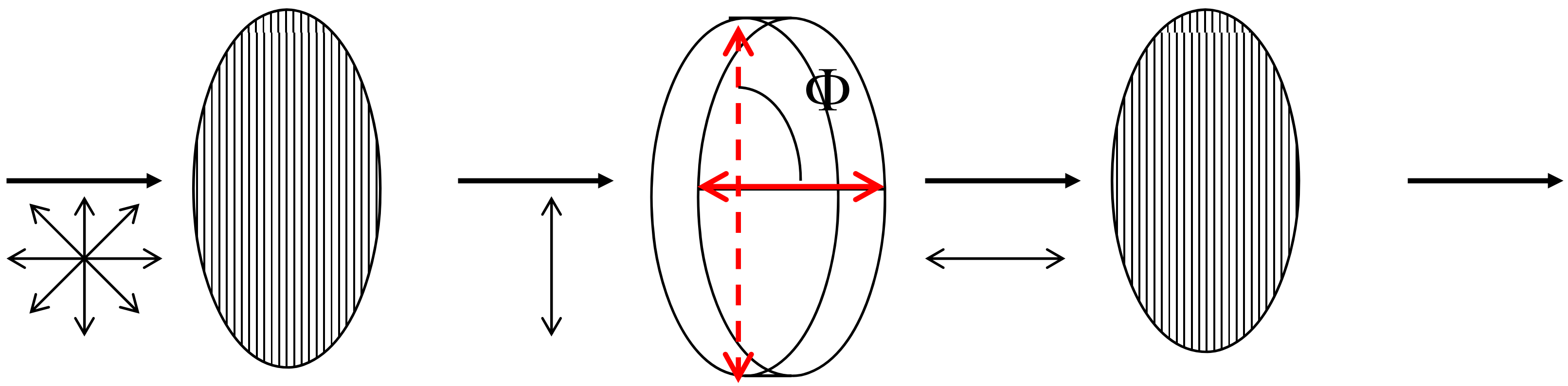


e-mode waveguiding



o-mode waveguiding

–TN cell with parallel polarizers (normally black mode)



$$V' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} R(-\Phi) \begin{pmatrix} \cos X - i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} & \Phi\dfrac{\sin X}{X} \\ -\Phi\dfrac{\sin X}{X} & \cos X + i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \Phi\dfrac{\sin X}{X} \end{pmatrix}$$

therefore

$$I = \left(\frac{1}{\sqrt{2}} \Phi \frac{\sin X}{X} \right)^2 = \frac{1}{2} \frac{\sin^2\left(\Phi\sqrt{1+u^2}\right)}{1+u^2}$$

where $u = \frac{2}{\lambda} \Delta nd$:mauguin parameter

Zero transmittance condition

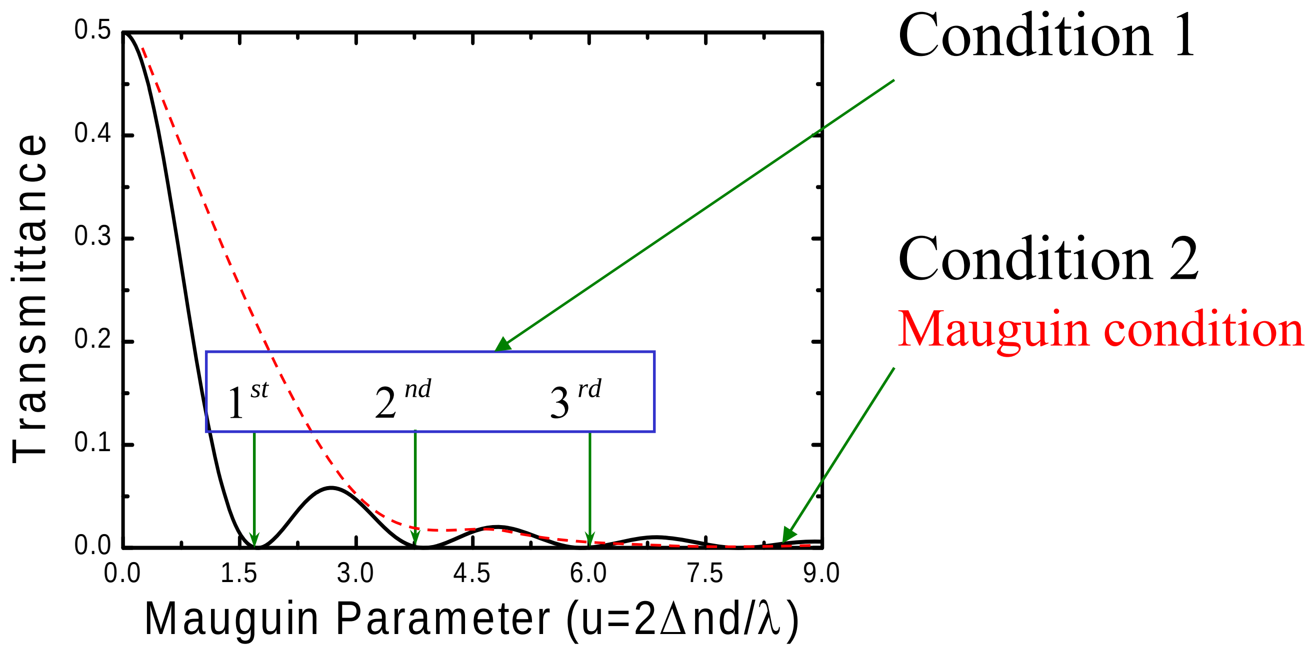
1. $\Phi\sqrt{1+u^2} = m\pi \ (m = 1, 2, 3, 4, \ldots)$

2. $u \approx \infty \ (\Delta nd >> \lambda) \longrightarrow \phi << \frac{2\pi}{\lambda} \Delta nd$

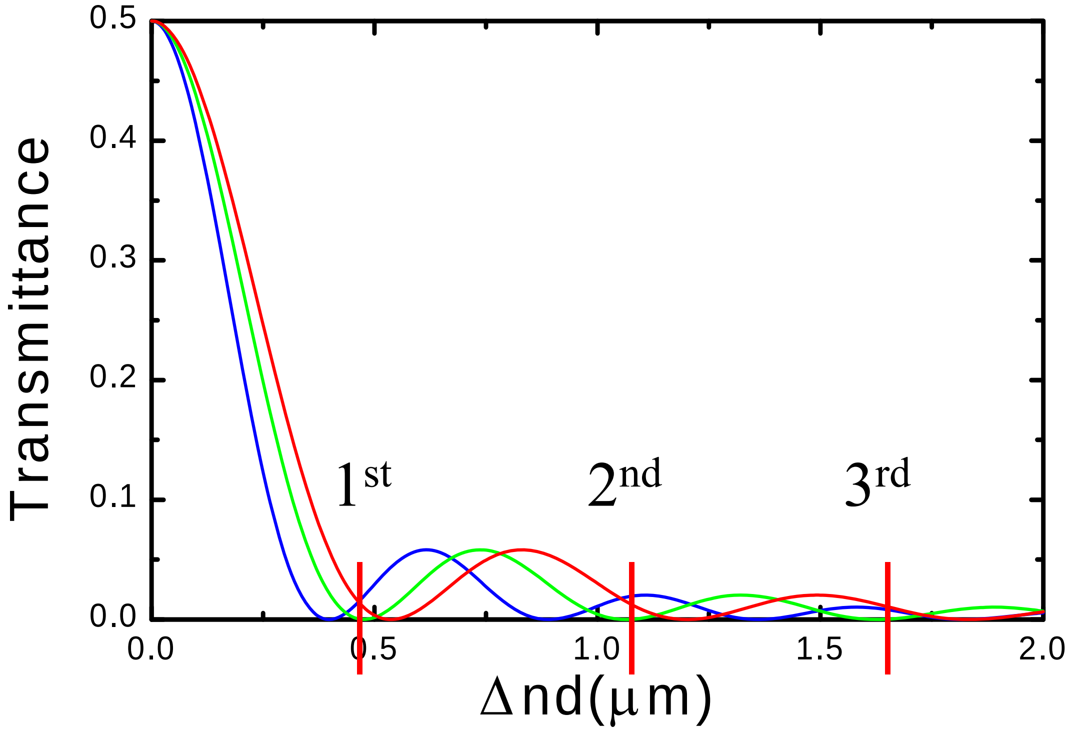
Mauguin condition

Zero transmittance condition

If $\Phi=\pi/2$



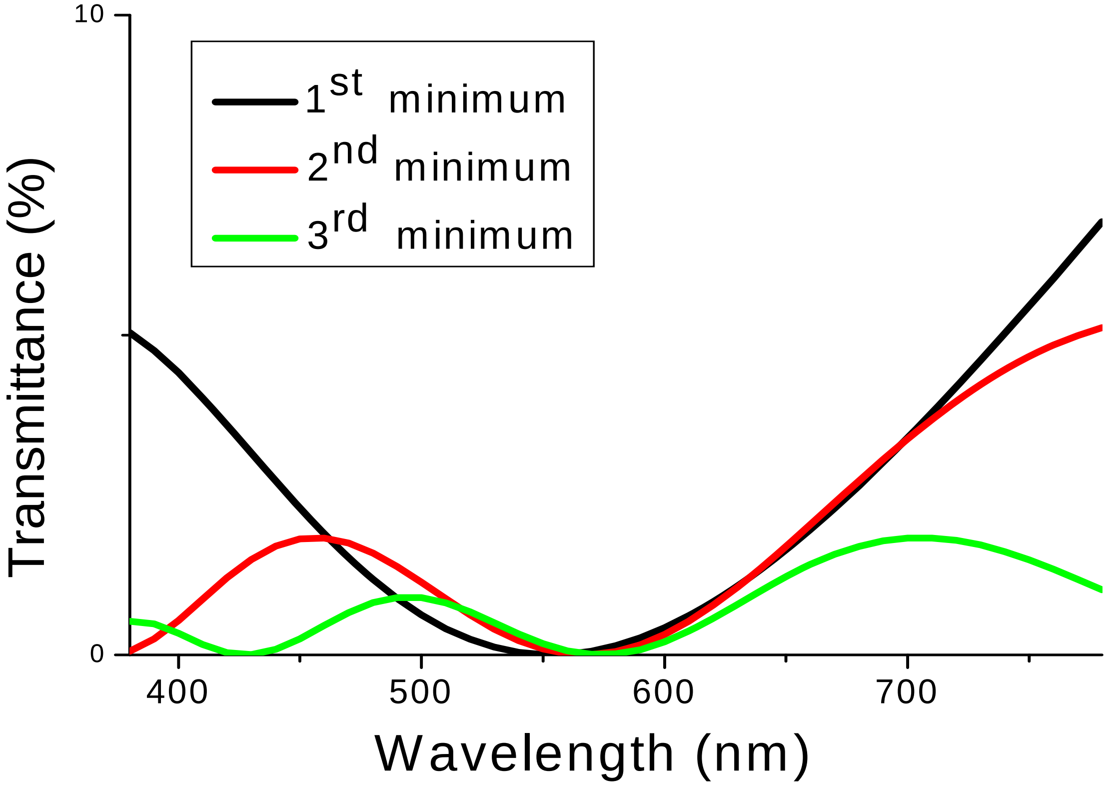
Phase dispersion in zero points



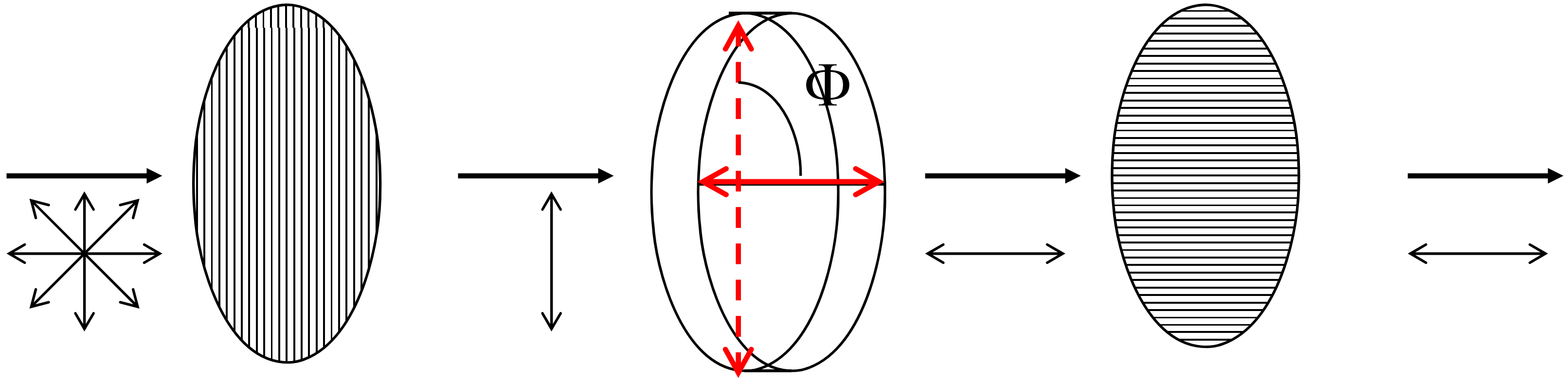
Phase dispersion makes a LC cell leak the light in the dark state

→ normally white mode in first minimum condition

Phase dispersion in the dark state



–TN cell with crossed polarizers (normally white mode)



$$V' = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} R(-\Phi) \begin{pmatrix} \cos X - i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} & \Phi\dfrac{\sin X}{X} \\ -\Phi\dfrac{\sin X}{X} & \cos X + i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} \begin{pmatrix} \cos X + i\dfrac{\Gamma}{2}\dfrac{\sin X}{X} \\ 0 \end{pmatrix}$$

therefore

$$I = \frac{1}{2}\left(1 - \frac{\sin^2\left(\Phi\sqrt{1+u^2}\right)}{1+u^2}\right)$$

where $u = \frac{2}{\lambda}\Delta nd$: mauguin parameter

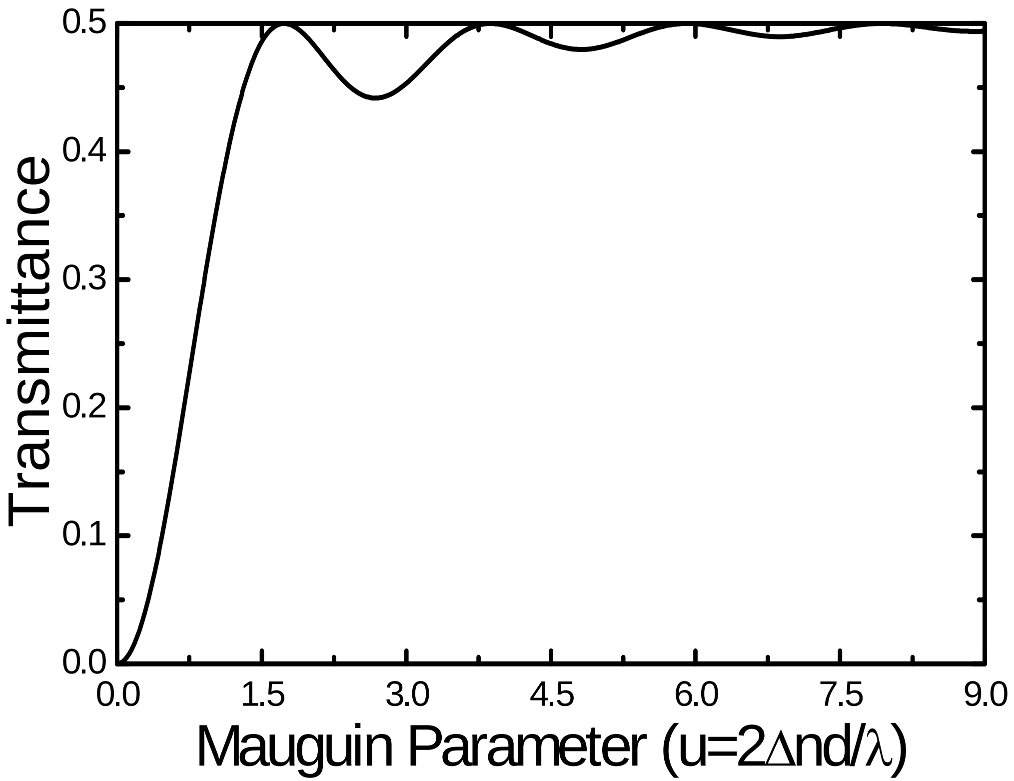
maximum transmittance condition

1. $\Phi\sqrt{1+u^2} = m\pi \ (m = 1, 2, 3, 4, \ldots)$

2. $u \approx \infty \ (\ \Delta nd \gg \lambda) \longrightarrow \phi \ll \frac{2\pi}{\lambda}\Delta nd$

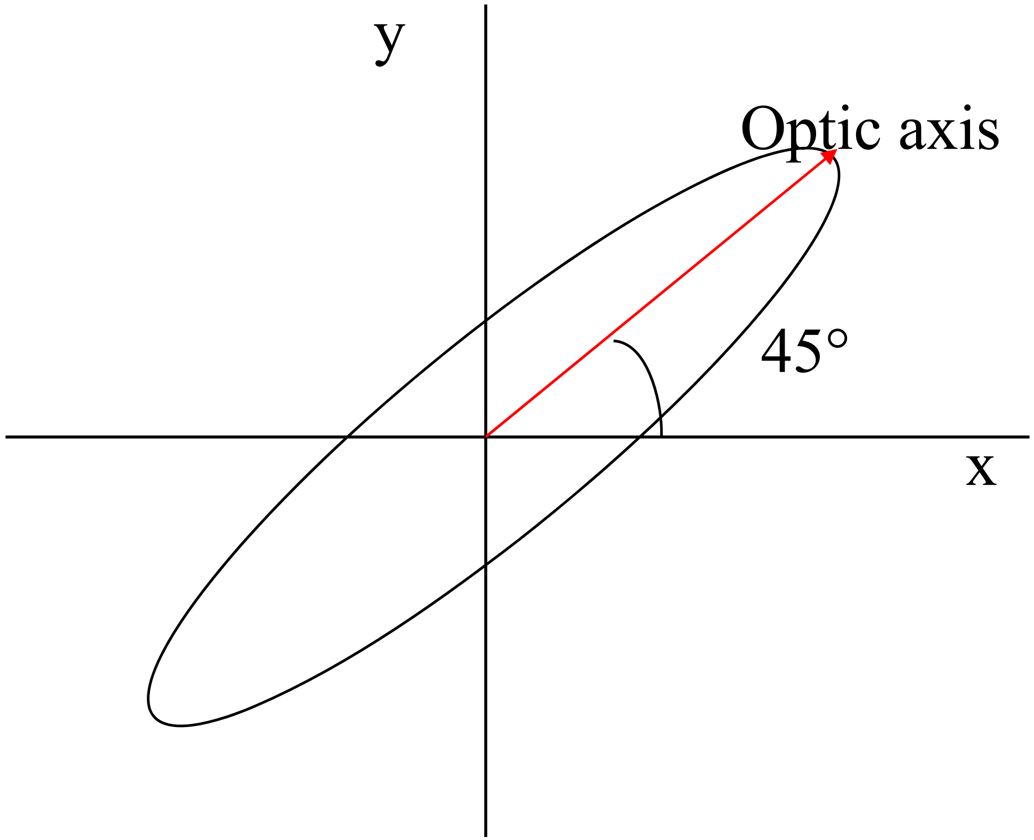
transmittance condition for N.W mode

If $\Phi=\pi/2$

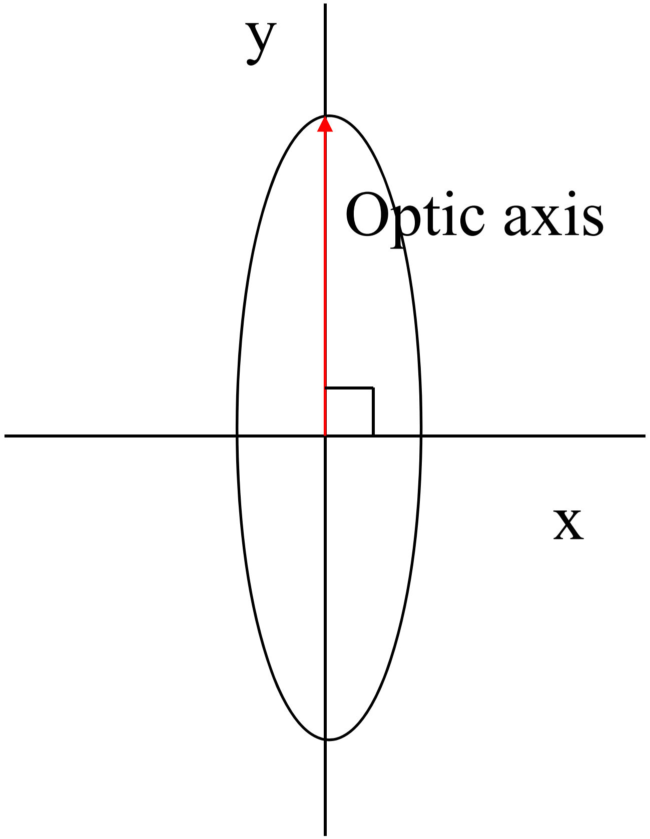


–TN cell and IPS cell with crossed polarizers

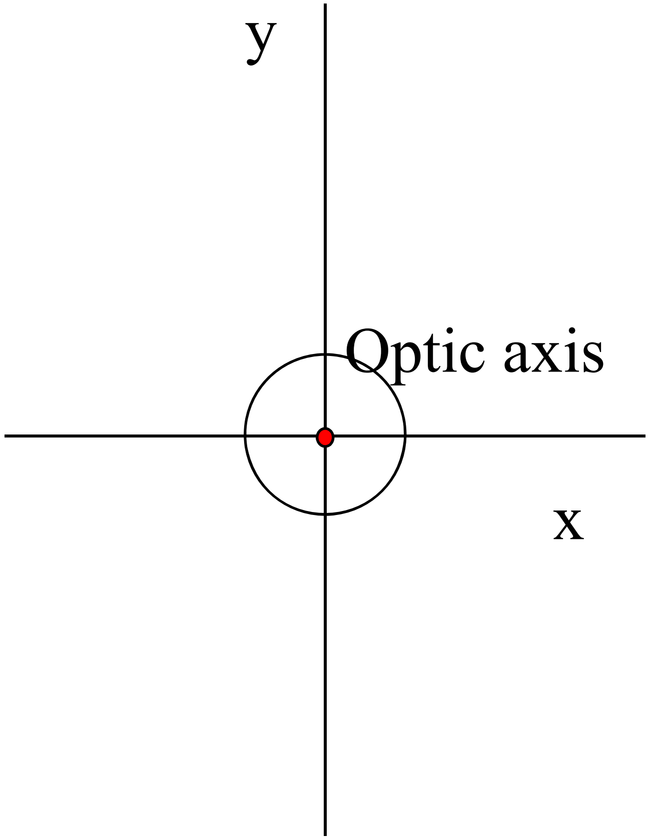
- Bright state : both of IPS and TN cell



- Dark state :



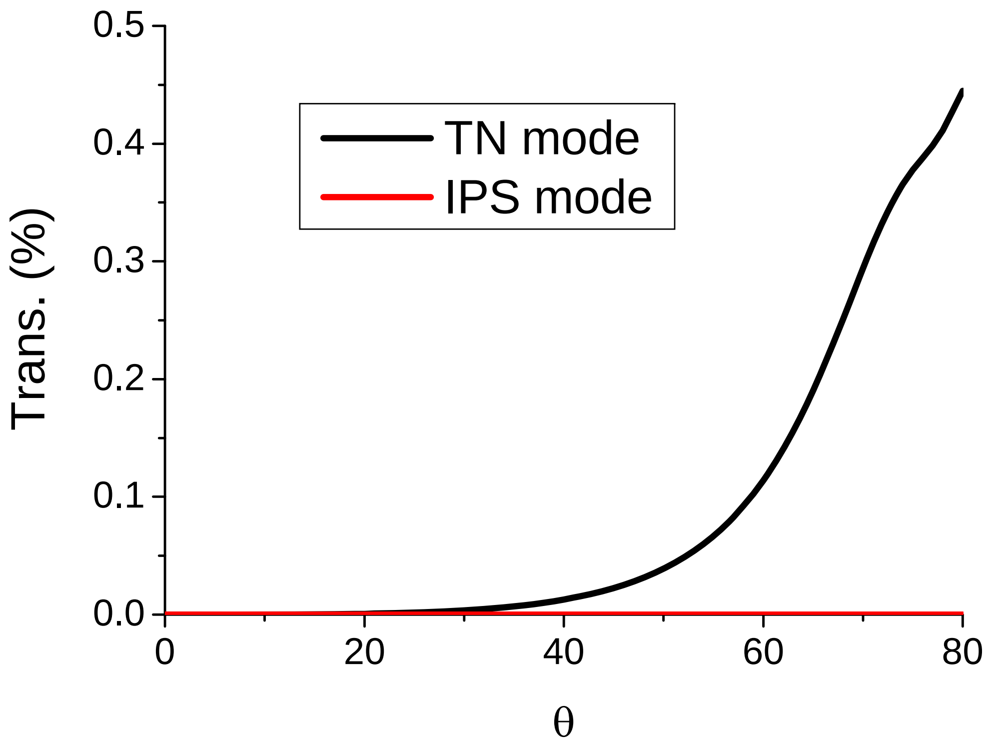
IPS mode



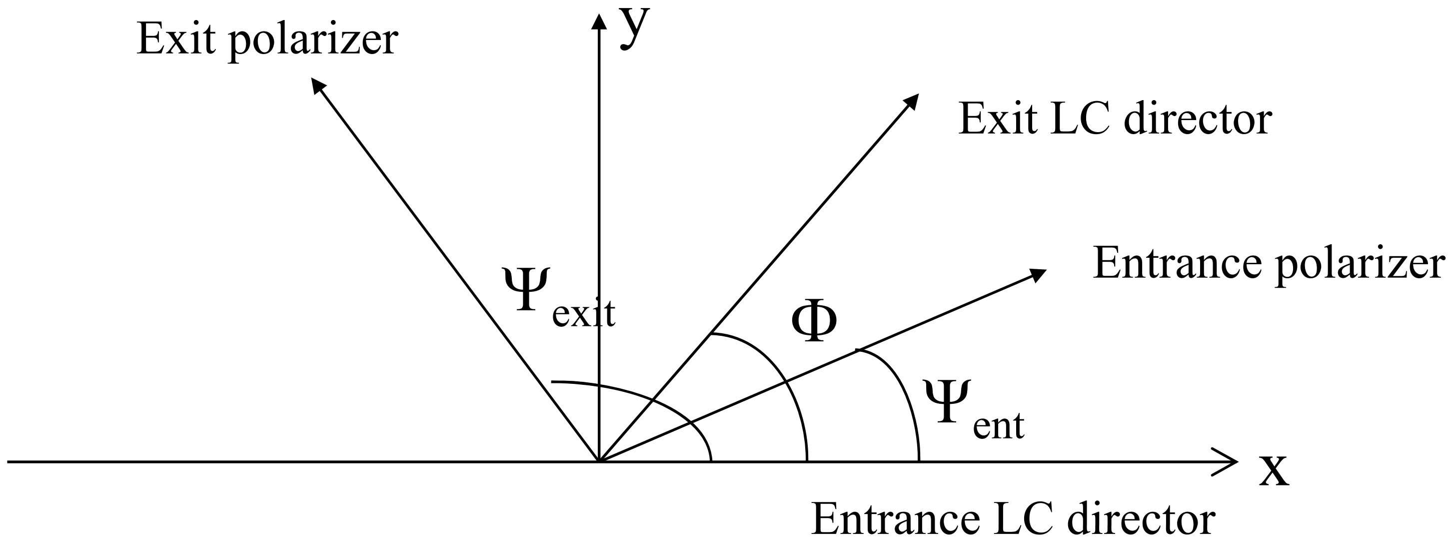
TN mode

– Optical transmission btn. TN cell and IPS cell in oblique incident direction

$$\frac{1}{n_e^2(\theta)} = \frac{\sin^2\theta}{n_e^2} + \frac{\cos^2\theta}{n_o^2}, \quad n_o^2(\theta) = n_o^2 \rightarrow \Delta n_{eff} = n_e(\theta) - n_o, \quad d_{eff} = \frac{d}{\cos\theta}$$



– Analytic solution for TN cell (1)



Input polarization
$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \begin{pmatrix} \cos\Psi_{ent} \\ \sin\Psi_{ent} \end{pmatrix}$$

Output polarization
$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} \cos\Psi_{exit} \\ \sin\Psi_{exit} \end{pmatrix}$$

Transmission $$T = \left| V' M V \right|$$

$$T = \cos^2(\alpha - \beta) - \sin \mathrm{X} \sin 2\beta \sin 2\alpha + \frac{\Phi}{2\mathrm{X}} \sin 2\mathrm{X} \sin 2(\alpha - \beta) - \Phi^2 \frac{\sin^2 \mathrm{X}}{\mathrm{X}^2} \cos 2\alpha \cos 2\beta$$

$$\alpha = \Psi_{ent}\ , \quad \beta = \Psi_{exit} - \Phi$$

therefore

$$T = \cos^2(\alpha + \beta) - \cos^2 \mathrm{X} \cos 2\beta \cos 2\alpha \left[\frac{\Phi}{\mathrm{X}} \tan \mathrm{X} - \tan 2\alpha \right] \left[\frac{\Phi}{\mathrm{X}} \tan \mathrm{X} + \tan 2\beta \right]$$

For maximum transmittance *T=1*

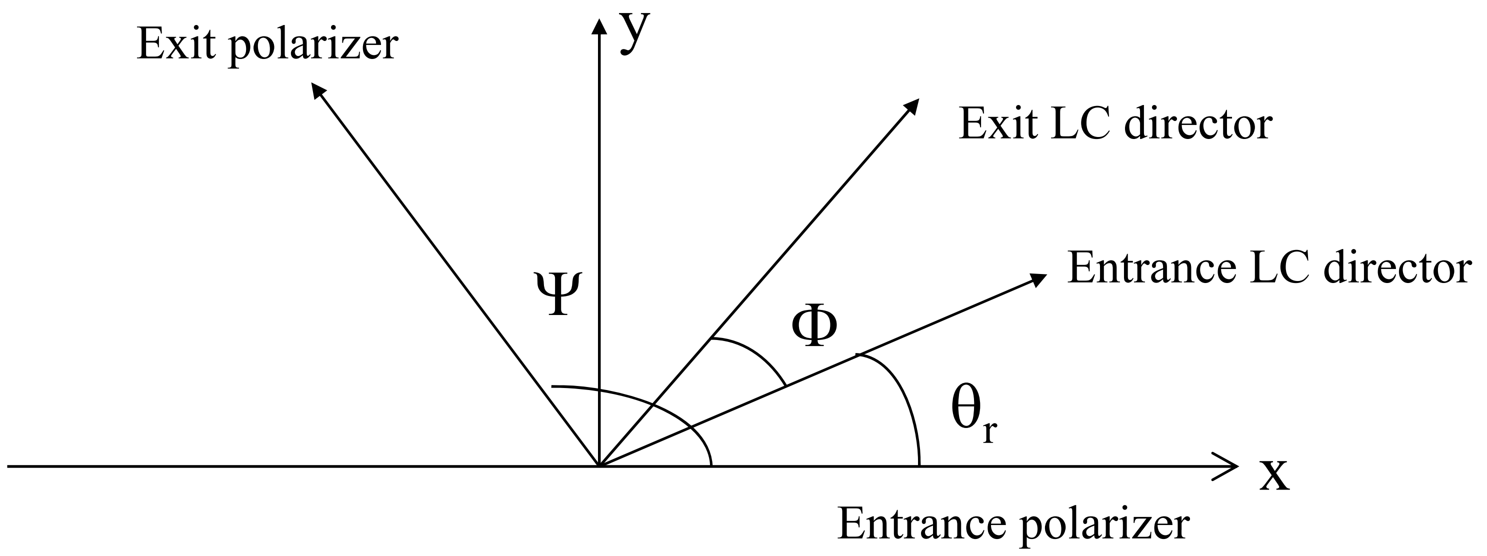
$$\frac{\Phi}{X}\tan X = \tan 2\alpha \;,\;\; \beta = -\alpha$$

$$\Psi_{exit} = \Phi - \Psi_{ent}$$

Decision of retardation

Decision of the polarizer angle

– Analytic solution for TN cell (2)



Input polarization

$$\begin{pmatrix} V_x \\ V_y \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Output polarization

$$\begin{pmatrix} V_x' \\ V_y' \end{pmatrix} = \begin{pmatrix} \cos\Psi \\ \sin\Psi \end{pmatrix}$$

Transmission $T = \left| V' M V \right|$

$$T = \left(\cos X \cos(\Phi - \Psi) + \frac{\Phi}{X} \sin X \sin(\Phi - \Psi) \right)^2 + \left(1 - \left(\frac{\Phi}{X} \right)^2 \right) \sin^2 X \cos^2 (2\theta_r + \Phi - \Psi)$$

This alignment is useful for measurement of *Φ* and *Δnd*

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